Introduction to Integration

Math 130 - Essentials of Calculus

14 April 2021

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$$\begin{aligned} F(x) &= x^2 + 5 \\ F(x) &= x^2 - \pi \\ F(x) &= x^2 - 2 \\ F(x) &= x^2 + 10000 \\ F(x) &= x^2 + 3\sqrt{2} \\ F(x) &= x^2 - \frac{4}{87} \end{aligned}$$

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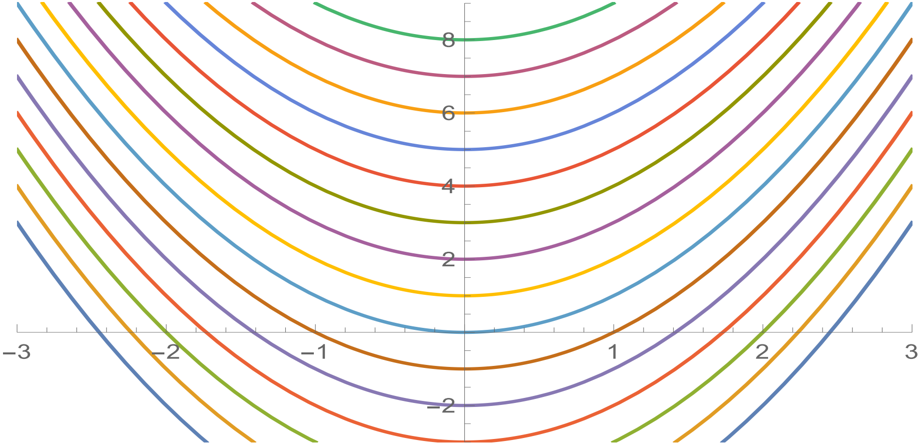
How do all of these relate to each other?

How Antiderivatives Differ

Here is a plot of various antiderivatives of $f(x) = 2x$.



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*Suppose that an antiderivative of $f(x)$ is given by $F(x)$. Then the **general antiderivative** of f is given by $F(x) + C$.*

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Note that we can use **any** antiderivative $F(x)$, though we usually take the simplest one in practice.

Examples

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6. $f(x) = 2x + 4$

Notation for Antiderivatives

As it is awkward to keep saying "the general antiderivative of $f(x)$ is $F(x) + C$," we use the notation of an **indefinite integral.** That is,

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- $\displaystyle\int kf(x)\,dx = k\int f(x)\,dx$

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5. $\int (x^3 - 3x + 1) dx$
6. $\int 3^x dx$